Weighted ω-Automata with Bounds

1 Context

In formal methods and theoretical computer science, weighted automata have long been a subject of intense study. These mathematical models extend traditional finite automata by associating input to weights, allowing for the quantitative analysis of system behaviors. See [4] for an introduction into the topic. Recent years have seen a growing interest in extensions of these automata, particularly in the context of energy constraints. Here the weight is interpreted as energy, which can be stored and accumulated. The energy is bounded from below in the sense that it must not drop below zero; it can also be bounded from above to model for instance a battery with a given capacity. These bounds can be interpreted as hard or weak bounds, depending on the semantics of the system and finding an energy feasible path in such automata is discussed in [2].

Building on these works, [5, 6] extend this approach to search for energy feasible paths in Büchi- and ω-automata. In this setting, finding and extracting such a path becomes significantly more involved as shown in Fig. 1. This complexity stems from the fact that, in order for a run to be valid, it needs to respect both the qualitative constraint given by the automaton (like the Büchi-condition shown here) and the quantitative constraint on the accumulated energy along the path. Due to the bounds on the energy, these constraints are tightly intertwined and con not be considered separately as in [3].

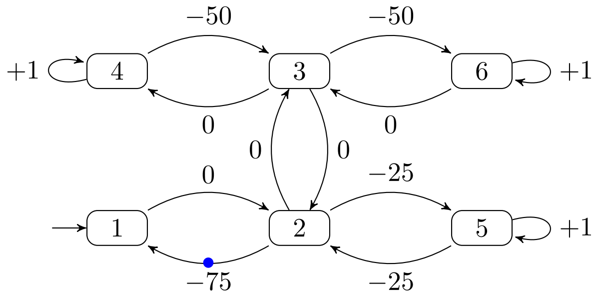

Figure 1: Non-trivial for ω-regular energy problems. Consider the energy to be bounded between 0 and 75. In order to satisfy the Büchi condition, every accepting path needs to take the accepting transition (The one carrying the blue dot) infinitely often. This transition however costs 75 energy, which gives and additional quantitative condition. To accumulate enough energy, we need to loop sufficiently often on the states 4, 5 and 6.

2 Objective

The objective of the internship is two-fold.
In a first step the goal is to get familiar with the theoretical concepts and our prototype implementation [1], create additional test and benchmark cases and to identify possible bottle-necks in the code. If the candidate is at ease or would like to sharpen his C++ skills, a rewrite in C++ and / or a tighter integration with spot is possible.
In a second step we wish to enlarge the capabilities in several ways and compare its performance to state-of-the-art tools solving similar problems. Indeed we hope that we can modify our approach in such a way that it is not only able to answer the emptiness problem (*Does a feasible path exist and return it if so*) but also the synthesis problem. In a synthesis setting, the states of the automaton are partitioned into two groups, those belonging to the *environment* and those belonging to the *system*. The environment and the system have to be interpreted as antagonistic players. The system tries to generate only runs that are both accepting and energy feasible. The environment on the other hand tries to prevent this. A game in this setting is an infinite succession of turns, where the player controlling the current state is allowed to freely chose one of the available successors states. Therefore the synthesis problem can be summed up as *Can the system ensure that it will win all possible games? Or are all plays energy feasible accepting runs, no matter the choices of the environment?*
In [1] it is shown that this synthesis problem can be solved in a symbolic fashion based on μ-calculus. Therefore the task is to pick up on our latest ideas and develop an efficient explicit algorithm to solve energy games and compare its relative performance.

3 Practical Information

The internship is supervised by Sven Dziadek and Philipp Schlehuber-Caissier (both assistant professors at Télécom Sudparis and members of SAMOVAR). It will preferably be located at the Télécom SudParis campus in Evry-Courcouronnes (9 rue Charles Fourier 91011 Evry-Courcouronnes) or possibly in Palaiseau (19 place Marguerite Perey 91120 Palaiseau).
If you are interested in this subject, do not hesitate to get in touch with us for further details: sven.dziadek@telecom-sudparis.eu and philipp.schlehuber-caissier@telecom-sudparis.eu.

4 Qualifications

- Foundations of automata theory and regular languages
- C++ and / or Python

References

[1] Gal Amram, Shahar Maoz, Or Pistiner, and Jan Oliver Ringert. Efficient algorithms for omega-regular energy games. In *International Symposium on Formal Methods*, pages 163–181. Springer, 2021.

[2] Patricia Bouyer, Uli Fahrenberg, Kim G Larsen, Nicolas Markey, and Jiří Srba. Infinite runs in weighted timed automata with energy constraints. In *Formal Modeling and Analysis*

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[1] `https://github.com/PhilippSchlehuberCaissier/wspot`

of Timed Systems: 6th International Conference, FORMATS 2008, Saint Malo, France, September 15-17, 2008. Proceedings 6, pages 33–47. Springer, 2008.

[3] Krishnendu Chatterjee and Laurent Doyen. Energy parity games. *Theoretical Computer Science*, 458:49–60, 2012.

[4] M. Droste, W. Kuich, and H. Vogler, editors. *Handbook of Weighted Automata.* EATCS Monographs in Theoretical Computer Science. Springer, 2009.

[5] Sven Dziadek, Uli Fahrenberg, and Philipp Schlehuber. ω-regular energy problems. *Formal Aspects of Computing*, 2022.

[6] Sven Dziadek, Uli Fahrenberg, and Philipp Schlehuber-Caissier. Energy büchi problems. In *International Symposium on Formal Methods*, pages 222–239. Springer, 2023.